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A CUT DISCONTINUOUS GALERKIN METHOD FOR THE LAPLACE-BELTRAMI OPERATOR

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In this talk, we consider a discontinuous Galerkin method for the Laplace-Beltrami problem on a smooth two dimensional surface embedded into a three dimensional background mesh consisting of tetrahedra. Since the surface is not aligned with the background mesh, we use the trace of discontinuous piecewise linears defined on the tetrahedra as trial and test functions in the discrete variational formulation. As the resulting linear system may be severely ill-conditioned due to possibly small intersections between the surface and the background mesh, consistent stabilizations terms are added to the original bilinear form. The proposed discretization scheme has optimal convergence properties and give raise to a well-conditioned discrete linear system independent of the intersection configuration. We conclude the presentation by illustrating the theoretical findings by a series of numerical experiments. As an application example, we consider the flow in porous media with fractures/faults, demonstrating the capability of our methods to compute the Darcy flow for a coupled surface-bulk problem when flow is present in both the crack surface and the bulk of the porous medium.

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